Elementary Maths for GMT

Exercises

Exercise 1. *Compute the length and the normalized vectors of the following vectors:*
(a) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ (c) $(2, 3, 2\sqrt{3})^T$

Exercise 2. *Compute the following operations:*
(a) $\begin{pmatrix} 1 \\ 2\sqrt{2} - 3 \\ 4 \end{pmatrix} + (3, \sqrt{3}, -\sqrt{5})^T$
(b) $\begin{pmatrix} 2 \\ -3 \\ 9 \end{pmatrix} - \sin(\frac{\Pi}{6}) \begin{pmatrix} 8 \\ 3.5 \\ -6 \end{pmatrix}$

Exercise 3. *Do the following dot product operations:*
(a) $(2, 5)^T \cdot (3, \frac{-6}{5})^T$
(b) $(3\sqrt{2}, 4, \sqrt{3})^T \cdot (\cos(\frac{\Pi}{4}), -1, \tan(\frac{\Pi}{6}))^T$

Exercise 4. *Using the dot product compute the angle between the vectors* $\begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix}$ *and* $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

Exercise 5. *Find the cosine of the angle between the vectors* $(2, 1, -2)^T$ *and* $(-2, 3, 6)^T$.

Exercise 6. *If* $\mathbf{u} = (4, 1, k)^T$ *and* $\mathbf{v} = (5, 1, -3)^T$ *are perpendicular, find* k.

Exercise 7. *If* $\mathbf{u} = (2, -6, k)^T$ *and* $\mathbf{v} = (-1, 3, 2)^T$ *are parallel, find* k.

Exercise 8. *What is the relation between the length of a vector and the dot product of the vector with itself?*

Exercise 9. *What do we know if* $v \cdot w = 0$ *for two non-null vectors* v *and* w*?*

Exercise 10. *Do the following cross product operations:*
(a) $(2, 5, 3)^T \times (-1, 3, 2)^T$
(b) $(1, 1, 2)^T \times (2, 0, 1)^T$
(c) $(1, -1, 2)^T \times (-2, 2, -4)^T$

Exercise 11. *Show that the cross product is not commutative.*

Exercise 12. *Find a nonzero vector which is perpendicular to both* $\mathbf{u} = (4, 1, 1)^T$ *and* $\mathbf{v} = (-1, 0, 2)^T$.

Exercise 13. *Find a nonzero vector which is perpendicular to both* $\mathbf{u} = (-2, -4, -6)^T$ *and* $\mathbf{v} = (3, -6, 1)^T$.

Exercise 14. *If the point* $(3, 1, k)$ *lies on the line* $(x, y, z)^T = (6, 2, 7)^T + t(3, 1, 5)^T$, *find* k.

Exercise 15. *Find a vector parallel to the line* $4x - 3y = 10$.

Exercise 16. *Give an implicit equation for the line which passes through the point* $(1, -1)$ *and is perpendicular to the line* $3x + 7y = 11$.

Exercise 17. *Find the point of intersection of lines ℓ_1 and ℓ_2 where:*
$\ell_1 : (x, y, z)^T = (0, -3, 6)^T + s(1, -1, -5)^T$
$\ell_2 : (x, y, z)^T = (-5, -7, 2)^T + t(3, 6, 2)^T$

Exercise 18. *Let ℓ be a line through the origin in $\mathbb{R}^2$, and let $n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$ be a normal vector for ℓ. Show* geometrically *that all points $p = (x_p, y_p)$ that lie on the line satisfy $n \cdot p^T = 0$.*

Exercise 19. *Let ℓ be a line in $\mathbb{R}^2$ that does* not *go through the origin, and let $n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$ be a normal vector for ℓ. Show* geometrically *that all points $p = (x_p, y_p)$ that lie on the line satisfy $n \cdot p^T - n \cdot p'^T = 0$, where p' is an arbitrary point on the line.*

Exercise 20. *Give a* parametric equation *for the plane V in $\mathbb{R}^3$ through the points $p_1 = (0, 7, 6)$, $p_2 = (8, 0, 8)$, and $p_3 = (12, 10, 0)$.*

The implicit equation of a plane in 3D has the form $ax + by + cz = d$, where $(a, b, c)^T$ is a *normal vector* of the plane.

Exercise 21. *Give an* implicit equation *for the plane V from the previous exercise.*

Exercise 22. *Give the implicit equation for the plane that passes through the point $(-1, 1, 3)$ and is parallel to vectors $(1, 4, -1)^T$ and $(2, -3, 1)^T$. What is the normal vector of the plane?*

Exercise 23. *Give both a parametric equation and an implicit equation for the plane V in $\mathbb{R}^3$ through the points $p_1 = (1, 2, 3)$, $p_2 = (4, 5, 6)$, and $p_3 = (2, 4, 5)$.*

Exercise 24. *What is the equation of the XY-plane?*

Exercise 25. *Find the point of intersection of the line $(x, y, z)^T = (-1, 2, 3)^T + t(2, 3, 4)^T$ with the plane $2x + y - z = 0$.*

Exercise 26. *Find the point of intersection of the line $(x, y, z)^T = (-3, 1, 2)^T + t(-1, 2, 3)^T$ with the plane $x - 3y + 2z = 4$.*

Exercise 27. *Do the following two parametric equations describe the same plane?*
$(x, y, z)^T = (1, 2, \frac{7}{5})^T + s(1, -1, -1)^T + t(2, 0, \frac{-4}{5})^T$

$(x, y, z)^T = (-2, 1, 2)^T + s(1, 2, \frac{4}{5})^T + t(0, 1, \frac{3}{5})^T$